

MAT 114 – 008
Fall 2007
Comprehensive Exam
Show all work.

1. Set up a system of linear equations to solve the following problem. You need not solve the system.

A student club sponsored a concert and charged admission: \$3 for students, \$5 for faculty, and \$8 for the general public. Ticket sales amounted to \$2542. Three times as many students bought tickets as faculty. The general public bought twice as many tickets as students did. How many tickets were sold to each group – students, faculty, and general public?

*2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system.

$$\begin{array}{rclcrcl} x & & & +9z & = & 12 \\ -2x & +2y & +z & & = & 3 \\ x & +2y & -3z & & = & 8 \end{array}$$

3. The augmented matrix of a system of linear equations (with unknowns x , y , z , and u) reduces to

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

3a. Write the system of equations that corresponds to this augmented matrix.

3b. Determine whether the system has a unique solution, no solution, or infinitely many solutions. Explain.

4. Write the initial tableau for the following linear programming problem. You need not solve the problem.

$$\text{Maximize } p = 20x + 30y.$$

subject to the following constraints.

$$2x + 10y \leq 80$$

$$6x + 2y \leq 72$$

$$3x + 2y \geq 6$$

$$x \geq 0$$

$$y \geq 0$$

5. The following is an initial tableau.

5a. Determine the pivot.

	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i>	<i>t</i>	<i>u</i>	<i>p</i>	
<i>s</i>	3	1	2	1	0	0	0	9
<i>t</i>	2	3	1	0	1	0	0	8
<i>u</i>	1	2	3	0	0	1	0	7
	-20	-12	-18	0	0	0	1	0

5b. Do this one pivot operation.

6. The following is an initial tableau.

6a. Determine the pivot.

	x	y	z	s	t	u	p	
s	1	0	1	-1	0	0	0	100
t	2	1	0	0	-1	0	0	50
u	0	1	1	0	0	-1	0	50
	2	2	3	0	0	0	1	0

6b. Do this one pivot operation.

7. The following is a final tableau. Determine the maximum value of p and the values of x , y , and z .

	x	y	z	s	t	u	p	
z	0	1	2	3	-1	0	0	90
x	6	3	0	-3	3	0	0	30
u	0	0	0	-9	-1	1	0	490
	0	0	0	12	0	0	3	600

*8a. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let p equal the amount of dietary fiber, x equal the number of servings of steak, and y equal the number of servings of cream of potato soup. You need not solve the problem.

In order to finish a term paper, Erin plans to work in her room all day on Saturday and eat all her meals at a nearby deli. She plans to eat only steak sandwiches and cream of potato soup. The nutritional contents, per serving, are:

	Steak sandwich	Cream of potato soup
calories	400	200
fat	12g	8.5g
dietary fiber	9g	5g

Erin wants to restrict her daily diet to no more than 2000 calories and no more than 65g of fat. How many servings of each food should she choose to maximize dietary fiber?

*8b. Write the initial tableau.

*8c. Determine the initial pivot. You need not do the pivot operation.

9. Translate the given system of linear equations into matrix form.

$$\begin{array}{rclcl} 3x & -2y & +7z & = & 6 \\ -2x & +y & & = & 4 \\ 6x & -5y & +8z & = & 4 \end{array}$$

10. Find the inverse of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -2 & 3 \end{bmatrix}$.

*11. $n(S) = 500$, $n(A) = 180$, $n(B) = 200$, $n(C) = 192$, $n(A \cap B) = 84$,
 $n(A \cap C) = 52$, $n(B \cap C) = 64$, and $n(A \cap B \cap C) = 38$. Find $n(A \cap B')$.

*12. $S = \{\text{cars in a dealer's lot}\}$.

$A = \{\text{cars in the lot that have automatic transmission}\}$.

$B = \{\text{cars in the lot with air conditioning}\}$.

$C = \{\text{cars in the lot with side air bags}\}$.

Using S , A , B , and C and set operations; describe the set of cars in the lot with automatic transmission and side air bags but without air conditioning.

13. In how many ways can a 6-letter security password be formed from the letters of the alphabet if no letter is repeated?

14. A manufacturer has 100 cell phones and wants to select 3 for testing. In how many ways can sets of 3 cell phones be selected for testing?

15. A company has 5 vacancies for its executive training program. 10 males and 10 females have applied for the training program. How many sets of 5 trainees can be selected if the vacancies must be filled by 2 men and 3 women?

16. In how many ways can 4-letter strings be made from the letters of GLACIER?

17. Among 500 sophomore students at a university, 320 are enrolled in an ECON 200, 225 are enrolled in MATH 109, and 140 are enrolled in both courses. What is the probability that a sophomore student is enrolled in exactly one of these two classes?

18. 2 light bulbs are selected at random from a lot of 24; 4 of the 24 bulbs in the lot are defective. What is the probability that of the 2 bulbs selected at least one is defective?

19 In a group of 200 students, 40 are taking English, 50 are taking mathematics, and 12 are taking both. What is the probability that a randomly selected student is taking English given that the student is taking mathematics?

20. A manufacturer buys an item from three subcontractors A, B, and C. 2% of the items furnished by A are defective, and A furnishes the manufacturer with 50% of the items. 5% of the items furnished by B are defective, and B furnishes the manufacturer with 30% of the items. 6% of the items furnished by C are defective, and C furnished the manufacturer with 20% of the items. Given that a defective item is selected, what is the probability that it came from manufacturer A?