

MAT 114 – 008  
Fall 2007  
Review for Comprehensive Exam  
Show all work.

1. Set up a system of linear equations to solve the following problem. You need not solve the system.

The total number of passengers riding a certain city bus during the morning shift is 1000. If the child's fare is \$.25 and the adult fare is \$.75 and the total revenue from fares in the morning shift is \$650, how many children and how many adults rode the bus during the morning shift?

2. For the following system of linear equations, set up the augmented matrix and use Gauss-Jordan reduction to solve the system.

$$\begin{array}{rclcrcl} x & +y & +z & = & 5 \\ x & +2y & +3z & = & 10 \\ 2x & +y & +z & = & 6 \end{array}$$

3. The augmented matrix of a system of linear equations (with unknowns  $x$ ,  $y$ ,  $z$ , and  $u$ ) reduces to

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

- 3a. Write the system of equations that corresponds to this augmented matrix.
- 3b. Determine whether the system has a unique solution, no solution, or infinitely many solutions. Explain.

4a. Graph the feasible set of the following set of linear inequalities.

$$\begin{aligned}x + 4y &\leq 360 \\2x + y &\leq 300 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

4b. Find the corner points of the feasible set.

4c. Is the feasible set bounded?

5. Write the initial tableau for the following standard maximum problem. You need not solve the problem.

$$\text{Maximize } p = 3x + 2y$$

subject to the following constraints.

$$\begin{aligned}2x + 3y &\leq 12 \\2x + y &\leq 8 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

6. The following is an initial tableau.

6a. Determine the pivot.

	$x$	$y$	$s$	$t$	$u$	$p$	
$s$	3	1	1	0	0	0	180
$t$	1	2	0	1	0	0	100
$u$	-2	2	0	0	1	0	40
$p$	-4	-12	0	0	0	1	0

6b. Do this one pivot operation.

7. The following is an initial tableau.

7a. Determine the pivot.

	$x$	$y$	$s$	$t$	$u$	$p$	
$s$	1	1	1	0	0	0	8
$t$	5	3	0	-1	0	0	21
$u$	1	3	0	0	-1	0	9
$p$	-8	-5	0	0	0	1	0

7b. Do this one pivot operation.

8. The following is a final tableau. Determine the maximum value of  $p$  and the values of  $x$  and  $y$ . (Show the values of  $x$ ,  $y$ , and  $p$  as fractions.)

	$x$	$y$	$s$	$t$	$u$	$p$	
$x$	9	0	4	-1	0	0	72
$y$	0	3	-1	1	0	0	18
$u$	0	0	-13	1	1	0	18
$p$	0	0	1	1	0	1	78

9a. Formulate the following linear programming problem; i.e., write the objective function and structural constraints. Let  $p$  equal return,  $x$  equal the amount invested in project A, and  $y$  equal amount invested in project B. You need not solve the problem.

A financier plans to invest up to \$500000 in two projects. Project A yields a return of 10% on the investment, whereas project B yields a return of 15% on the investment. Because the investment in project B is riskier than the investment in project A, she has decided that the investment in project B should not exceed 40% of the total investment. How much should the financier invest in each project in order to maximize the return on her investment? What is the maximum return?

9b. Write the initial tableau.

10. Translate the given system of linear equations into matrix form.

$$\begin{aligned} 3x - 5y + 4z &= 10 \\ 4x + 2y - 3z &= -12. \\ -x &+ z = -2 \end{aligned}$$

11. Find the inverse of  $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$ .

12.  $n(S)=100$ ,  $n(A)=28$ ,  $n(B)=30$ ,  $n(C)=34$ ,  $n(A \cap B)=10$ ,  
 $n(A \cap C)=10$ ,  $n(B \cap C)=15$ , and  $n(A \cap B \cap C)=5$ . Find  $n(A \cap (B \cup C))$ .

13.  $S = \{\text{students in a certain business college}\}$ .

$A = \{\text{students who have taken a course in accounting}\}$ .

$B = \{\text{students who have taken a course in economics}\}$ .

$C = \{\text{students who have taken a course in marketing}\}$ .

Using  $S$ ,  $A$ ,  $B$ , and  $C$  and set operations; describe the set of students who have had at least one of the three classes.

14. The State of California issues license plates with the identification string consisting of one letter of the alphabet followed by 3 digits followed by 3 letters of the alphabet. How many different license plates may be issued using this configuration?

15. How many arrangements of 3 people seated along one side of a table are possible if there are 8 people to choose from?

16. From a set of 7 mathematics books, 5 literature books, and 9 science books; in how many ways can a student select 2 from each set?

17. A club has 9 seniors, 16 juniors, 14 sophomores, and 10 freshmen. In how many ways can 2 representatives be selected if the selection must be composed of either a senior and a junior or a junior and a sophomore?

18. Three instructors, Ms. Busby, Mr. Butler, and Mrs. Hutchinson, assign projects in their precalculus classes. Some students use graphing calculators with their project, and some do not. The following table summarizes the number of students in each type of project:

	Busby	Butler	Hutchinson	Total
Calculator	13	11	17	41
No calculator	15	16	9	40
Total	28	27	26	81

18a. What is the probability that a student uses a graphing calculator?

18b. What is the probability that a student is Ms. Busby's student and does not use a graphing calculator?

18c. What is the probability that a student is a student of Mr. Butler?

18d. What is the probability that a student is uses a graphing calculator given that the student is a student of Mr. Hutchinson?

18e. What is the probability that a student is a student of Mr. Hutchinson given that the student uses a graphing calculator?

19. 2 people are selected at random from a group of 12 Republicans and 10 Democrats. What is the probability that 1 is a Republican and 1 is a Democrat?

20. A standardized reading test was given to fourth- and fifth-grade classes at an elementary school. A summary of the results is the following:

	Scoring Below Grade Level	Scoring at Grade Level	Scoring Above Grade Level	Total
4th grade	120	342	216	678
5th grade	105	324	98	527
Total	225	666	314	1205

A student is selected at random from this group. Find the probability that the student:

20a. Is a fourth-grade student.

20b. Is a fourth-grade student working above grade level.

20c. Is a fifth-grade student scoring at or above grade level.

20d. Is a fourth-grade student given that the student is scoring below grade level.

20e. Is scoring below grade level given the student is in the fourth grade.