

MAT 222 - 001
Fall 2007
Test One
Show all work.

On the integration problems (1 and 2), show the details of each step.

1. $\int_{-3}^{\infty} \frac{dx}{(x+4)^{3/2}}.$

2. $\int_1^3 \frac{dx}{\sqrt{3-x}}$

3. Use Simpson's rule with $n = 10$ to approximate $\int_2^4 \sqrt{x^4+1} dx.$

4. Write the first five terms of the sequence

$$a_n = \frac{3n+1}{2n+4}.$$

Determine whether the sequence converges or diverges. If it converges, find its limit.

5. Write the first five terms of the sequence

$$\left\{ 5 - \frac{9}{n^2} \right\}.$$

Determine whether the sequence converges or diverges. If it converges, find its limit.

6. Write the series $\frac{1}{1} - \frac{2^2}{2 \times 1} + \frac{3^3}{3 \times 2 \times 1} - \frac{4^4}{4 \times 3 \times 2 \times 1} + \dots$ in \sum -notation.

Determine whether each of the following series converges or diverges. If the series is a convergent geometric series, find its sum. State the test you are using and show all details of the test. Always verify the conditions needed for the test to be valid.

7. $\frac{0}{1} - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \dots$

8. $\sum_{k=1}^{\infty} \frac{1}{k(k+3)}$.

9. $\sum_{k=1}^{\infty} k^{-1/3}$.

10. $\sum_{k=1}^{\infty} \frac{7 \times 3^k}{11^k}$.

11. $\sum_{k=5}^{\infty} \frac{1}{\sqrt{k-4}}$.

12. $\frac{64}{49} + \frac{8}{7} + 1 + \frac{7}{8} + \frac{49}{64} + \frac{343}{512} + \dots$

$$13. \sum_{k=1}^{\infty} \frac{8+2^k}{5^k}.$$

$$14. \cos \frac{1}{2} + \cos \frac{1}{3} + \cos \frac{1}{4} + \cdots$$

$$15. \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}.$$